Exercise 40

For the following exercises, find $(f \circ g)$ and the domain for $(f \circ g)(x)$ for each pair of functions.

$$f(x) = \frac{1}{x + 3}, \quad g(x) = \frac{1}{x - 9}$$

Solution

Calculate $(f \circ g)(x)$ by plugging the formula for $g(x)$ in where x is in the formula for $f(x)$.

$$(f \circ g)(x) = f(g(x))$$

$$= \frac{1}{\frac{1}{x-9} + 3}$$

$$= \frac{1}{\frac{1}{x-9} + 3} \times \frac{x - 9}{x - 9}$$

$$= \frac{x - 9}{1 + 3(x - 9)}$$

$$= \frac{x - 9}{1 + 3x - 27}$$

$$= \frac{x - 9}{3x - 26}$$

The denominator of this rational function cannot be zero at any step.

$$x - 9 \neq 0 \quad \text{and} \quad 3x - 26 \neq 0$$

Solve for x.

$$x \neq 9 \quad \text{and} \quad x \neq \frac{26}{3}$$

Therefore, the domain of $(f \circ g)(x)$ is

$$\left(-\infty, \frac{26}{3}\right) \cup \left(\frac{26}{3}, 9\right) \cup (9, \infty).$$